

A Mathematical Model to Predict the Strength of Aluminum Alloys Subjected to Precipitation Hardening

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A number of alloys, notably most of the aluminum alloys, can be heat treated by aging. This aging due to time-dependent precipitation hardening increases the strength and hardness as well as modifying other mechanical properties. Precipitation hardening has been a popular strengthening mechanism for many decades; therefore, extensive information is available in literature about the precipitation-hardening response of various series of aluminum alloys. The age-hardening response of these alloys is usually represented in graphical form as plotted between property changes and aging time for different temperatures. In designing a suitable precipitation-hardening strategy, one can refer to these graphs. However, for automatic control of aging furnaces, as well as for decision making regarding optimal selection of aging conditions (time/temperature combination), it is desirable to express these relationships in a formal mathematical structure. A mathematical model is developed in this article for widely used heat treatable aluminum alloys used in the extrusion industry. This model is a condensed representation of all $\sigma = f(T, t)$ curves in different series of aluminum alloys, and the parameters of this model characterize the various compositions of the alloys in the series.

Keywords age hardening, aging, aluminum, aluminum alloys, aluminum extrusions, cooling rate, extrudable alloys, precipitation hardening, regression, strength

1. Introduction

In certain alloys, precipitation of solute-rich particles occurs from supersaturated solid solution (which is prepared by solution treatment) after its quenching. This leads to an increase in the strength of the alloy, termed precipitation hardening, age hardening, or aging. The best combination of mechanical properties is achieved when a uniform dispersion of fine solute particles can be obtained. If the particles become coarser or precipitate at the grain boundaries, the mechanical properties are impaired (Ref 8). The two major factors that influence the mechanical properties as a result of age hardening are aging time and aging temperature. Conventionally, the age-hardening response of the alloys is graphically represented in the form of "aging curves." These aging curves represent the aging-time/mechanical-property behavior of the material at a constant aging temperature and tend to follow a skewed, bell-shaped profile. These curves indicate an initial enhancement and a later degradation of mechanical properties that is consistent with the precipitation of fine, uniformly dispersed particles and then their coarsening with increasing aging time.

The objective of this work is to formulate a mathematical model representing the strength dependence of aged aluminum alloys on aging parameters (aging time and aging temperature).

2. Available Data

In order to develop a mathematical model, experimental data are required that relate the mechanical properties with the aging parameters (aging time and temperature). Although a number of aluminum alloys have been developed for various applications, this work is directed toward heat treatable aluminum alloys, which are widely used in the extrusion industry. Approximately 60 to 70% of the extrusions sold in the United States and perhaps worldwide are from the alloy family AA6063/AA6060/AA6061/AA6005A (Ref 1). Of these alloys, AA6063 is the most widely used as a general-purpose alloy. Alloy AA6060 is similar in composition as well as in mechanical properties, to AA6063 (Ref 2). AA6061 is also widely used for extrusion and has a sizable consumption in industry (Ref 3). The aging data for AA6063—a popular structural, heat treatable aluminum alloy—are obtained from literature (Ref 4) through a major manufacturer of aluminum products in the Middle East. These data are currently being used in the commercial aging process by this manufacturer and are used for the task of mathematical modeling. The strength is measured in kg/mm^2 (9.81 MPa).

The aging data for alloy AA6061 are taken from Ref 5. These data are for general T6 condition, indicating that specimens were solution treated at about 530 °C, quenched, and artificially aged at various temperatures. The strength is measured in MPa.

Experimental aging data of alloy AA6069, which is a new alloy produced for improved mechanical properties, are also taken from literature (Ref 6). The available data are for hot extrusion, and strength is measured in MPa.

3. Mathematical Modeling

The available data are in graphical form, indicating the effect of aging time (in hours) on yield strength and tensile strength (kg/mm^2 or MPa) at different aging temperatures. The data are plotted on a peculiar log scale (for one curve it is of base 10, for the second curve it is of base 2, and for the third curve it is linear). Digitized data were obtained from the aging curves and were tabulated on a linear time scale.

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Initial observation of the data and the understanding of the process indicates that the functional relationship should represent the following characteristics:

- The model should be a function of time and temperature.
- The function should initially indicate an increasing and then a decreasing behavior.
- The function should give a certain value of mechanical property being modeled at the inception of the aging process.

In general, the model should have the following form:

$$\sigma = \sigma_o + f(t, T) \qquad \text{(Eq 1)}$$

where σ is the tensile strength, σ_0 is the initial tensile strength at the inception of the aging process, and t and T are time and temperature, respectively.

A number of different mathematical models were explored for their adequacy to represent the second term in the right-hand side of Eq 1, that is, the function $f(t, T)$. The suitability of the fitted equation (model) was assessed using several statistical measures; the main criterion was the coefficient of determination (r^2), which gives the comparison of predicted and actual values. F-statistic and P-values are two other criteria that can be used to assess the model.

Based on these statistical measures, almost all of these models were rejected, mainly because of the poor (r^2) values indicating the poor prediction from the models. Considering the resemblance of the aging curves with the general shape of the skewed distributions, another model was proposed. This model, based on the Weibull distribution (Ref 7), has the following form:

$$f(t) = A \cdot t^B \cdot \exp\,(C \cdot t^n) \qquad \text{(Eq 2)}$$

This function does represent an initial increasing and then a decreasing trend. However, the model in this form is inadequate because only one independent variable is involved. The model represents the age-hardening response at a certain aging temperature, while aging time is varied. This model, however, can be extended to include the temperature as a variable if the coefficients in the model are considered temperature dependent and explored using a double (or two-stage) regression technique. The procedure for this technique is:

- In the first step (stage 1), regression of data at each individual aging temperature was separately carried out to explore the predictive model for a single temperature. This resulted in a set of equations for both yield and tensile strength as a function of time at a constant temperature.
- In the subsequent step (stage 2), regression of the coefficients obtained in the first step was carried out to determine their dependence on temperature.
- The final form of the model is:

Table 1 Regression models for yield and tensile strength of AA6063

Temperature (T), °C	Model (yield strength)	Model (tensile strength)
130	$100 + 14.9052\,t^{0.949} \exp[-0.1318\,t^{0.8}]$	$200 + 25.2704\,t^{0.1766} \exp[0.01967\,t^{0.8}]$
155	$100 + 24.0801\,t^{0.777} \exp[-0.0391\,t^{0.8}]$	$200 + 30.6367\,t^{0.3065} \exp[0.01781\,t^{0.8}]$
170	$100 + 53.3193\,t^{1.138} \exp[-0.2212\,t^{0.8}]$	$200 + 42.7264\,t^{0.5671} \exp[-0.0777\,t^{0.8}]$
185	$100 + 85.5595\,t^{1.384} \exp[-0.3910\,t^{0.8}]$	$200 + 55.3480\,t^{0.6122} \exp[-0.1506\,t^{0.8}]$
200	$100 + 190.574\,t^{1.387} \exp[-0.6728\,t^{0.8}]$	$200 + 100.850\,t^{0.9497} \exp[-0.5374\,t^{0.8}]$
220	$100 + 940.119\,t^{2.363} \exp[-1.9840\,t^{0.8}]$	$200 + 414.528\,t^{1.6358} \exp[-1.8826\,t^{0.8}]$

Table 2 Regression models for yield and tensile strength of AA6061

Temperature (T), °C	Model (yield strength)	Model (tensile strength)
120	$150 + 3.4975\,t^{0.6322} \exp[-0.0888\,t^{0.36}]$	$250 + 7.2436\,t^{0.1766} \exp[-0.1744\,t^{0.36}]$
150	$150 + 19.8943\,t^{0.4777} \exp[-0.1201\,t^{0.36}]$	$250 + 17.46\,t^{0.3065} \exp[-0.367\,t^{0.36}]$
170	$150 + 47.73\,t^{0.9856} \exp[-0.6944\,t^{0.36}]$	$250 + 40.7175\,t^{0.5671} \exp[-0.7692\,t^{0.36}]$
205	$150 + 1754.839\,t^{1.2447} \exp[-2.692\,t^{0.36}]$	$250 + 1185.441\,t^{0.6122} \exp[-3.3240\,t^{0.36}]$
230	$150 + 15{,}457.86\,t^{1.3056} \exp[5.0598\,t^{0.36}]$	$250 + 10{,}773.22\,t^{0.9497} \exp[-6.7474\,t^{0.36}]$

Table 3 Regression models for yield and tensile strength of AA6069

Temperature (T), °C	Model (yield strength)	Model (tensile strength)
160	$300 + 17.33\,t^{0.909} \exp[-0.0499\,t^{0.9}]$	$400 + 47.658\,t^{0.1984} \exp[0.000072\,t^{0.9}]$
171	$300 + 20.5198\,t^{1.1769} \exp[-0.0963\,t^{0.9}]$	$400 + 13.16\,t^{0.9508} \exp[-0.0649\,t^{0.9}]$
182	$300 + 82.8775\,t^{0.3977} \exp[-0.0388\,t^{0.9}]$	$400 + 62.535\,t^{0.2129} \exp[-0.03363\,t^{0.9}]$
193	$300 + 140.171\,t^{0.0504} \exp[-0.0123\,t^{0.9}]$	$400 + 82.1723\,t^{-0.1201} \exp[-0.00548\,t^{0.9}]$

$$\sigma = \sigma_0 + A(T) \cdot t^{B(T)} \cdot \exp(C(T) \cdot t^n) \qquad \text{(Eq 3)}$$

where $A(T)$, $B(T)$, and $C(T)$ are temperature-dependent functions and σ_0 and n are constants.

4. Results and Discussion

The graphical data was digitized and tabulated for $\sigma - \sigma_0$. The data for each aging temperature were then regressed to obtain the coefficients of Eq 2. In stage 1, the regression was initially carried out for different values of n, and then n was forced to be a constant. The results from stage 1 regression for yield strength and tensile strength are compiled in Tables 1 to 3.

The main statistical criterion for the measure of the model performance, the values of (r^2) were found to be more than 0.9 for most of these curves. The predicted models were then compared with the actual data, and the comparisons for AA6063 are shown in Fig. 1 and 2 for yield and tensile strength, respectively. It is evident from these results that the coefficients A, B, and C are a function of aging temperature (T), while σ_0 and n are constants.

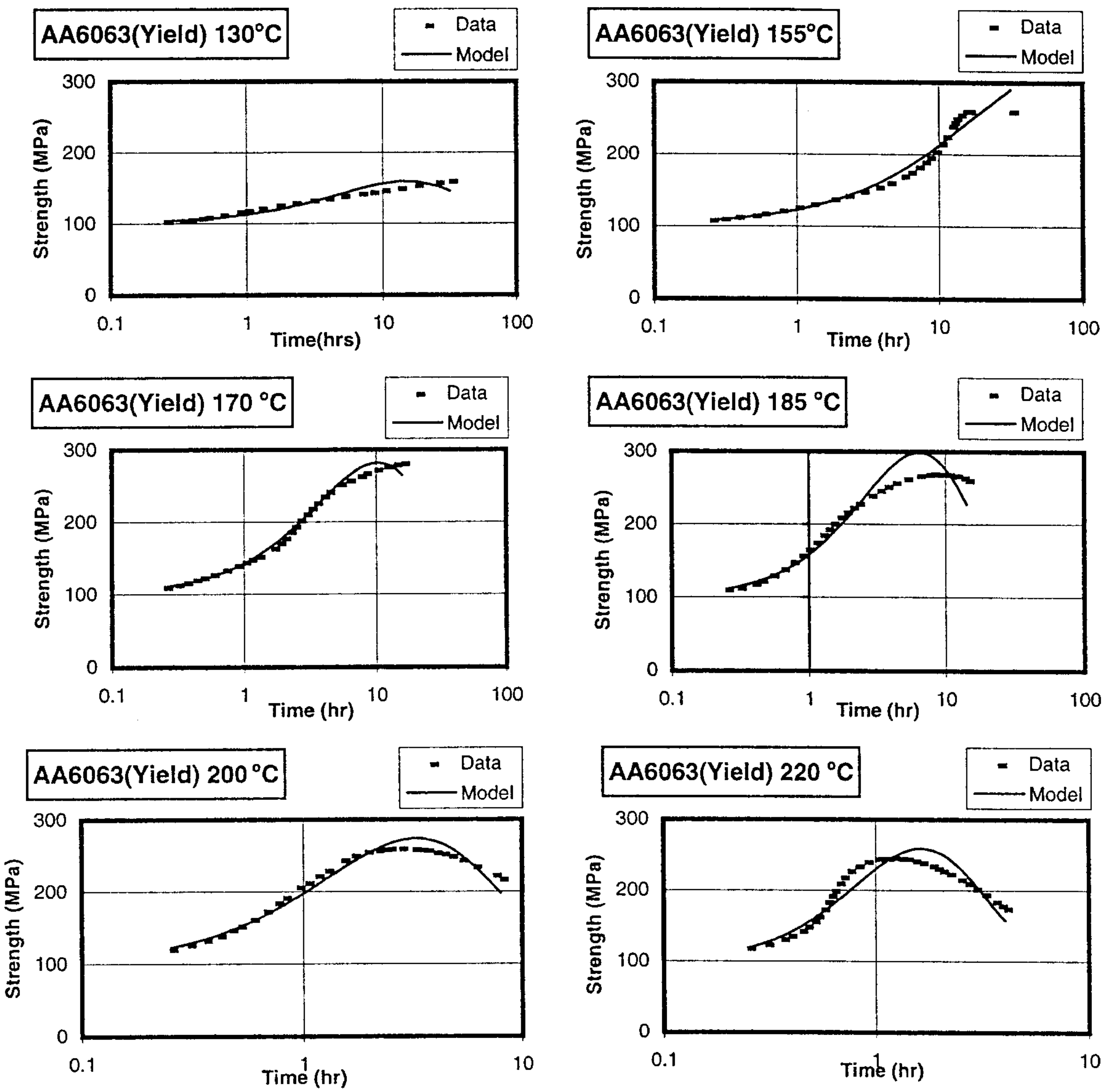

Fig. 1 Comparison of data and model after stage 1 regression (yield strength)

Once the values of these temperature-dependent coefficients are available from the stage 1 regression, their functional dependence on temperature can be obtained, again by regression. Based on the physics of the problem, the dependence of yield and tensile strengths on temperature was initially modeled to be related to four different terms involving temperature. These terms are the simple T term, indicating the direct dependence of the process on aging temperature, a reciprocal T term representing the Arrhenius nature of the model of the aging process, the square of T, and square root of T terms check the variation of coefficients with temperature. In general, the coefficients $A(T)$, $B(T)$, or $C(T)$ can be modeled as:

$$Y(T) = y_o + y_1 T + y_2 (1/T) + y_3 \sqrt{T} + y_4 T^2 \quad \text{(Eq 4)}$$

where $Y(T)$ represents a coefficient of the model. The significance of these terms was tested using analysis of variance (ANOVA), and all of these terms were found to be significant for the model for a confidence level of 90% and above. Additionally, the values of r^2 for stage 2 regression were found to be more than 0.95. Thus, in the final form of the model:

$$\sigma - \sigma_o = A(T) \cdot t^{B(T)} \exp(C(T) \cdot t^n)$$

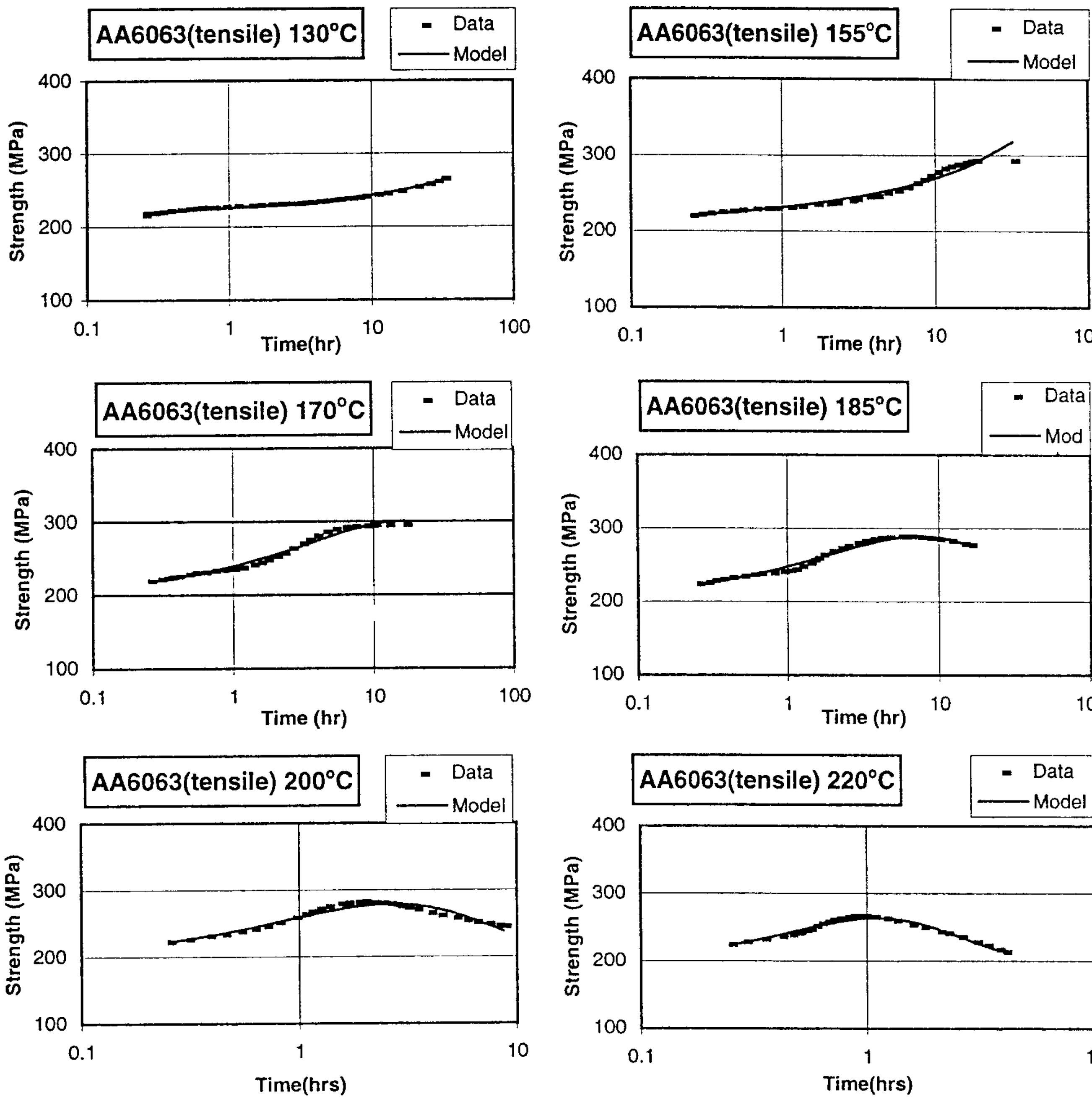

Fig. 2 Comparison of data and model after stage 1 regression (tensile strength)

The parameters $A(T)$, $B(T)$, and $C(T)$ are given as:

$$A = a_o + a_1 T + a_2 (1/T) + a_3 \sqrt{T} + a_4 T^2$$

$$B = b_o + b_1 T + b_2 (1/T) + b_3 \sqrt{T} + b_4 T^2$$

$$C = c_o + c_1 T + c_2 (1/T) + c_3 \sqrt{T} + c_4 T^2 \qquad \text{(Eq 5)}$$

The values of coefficients a_i, b_i, and c_i (where i = 0, 1, 2, 3, 4) and σ_o are given in Table 4 for yield strength and in Table 5 for tensile strength, and the value of n was 0.8, 0.36, and 0.9 for AA6063, AA6061, and AA6069, respectively, as previously stated. The comparisons of the models and data for yield strength and tensile strength are presented in Fig. 3 to 8 for the alloys used in this work. This is an important development if computer-controlled aging is to be used; the above relationships can be used to effectively develop such controls. Additionally, if due to some reason the aging temperatures for which the experimental curves are available cannot be achieved, this model can be used to predict the resulting mechanical properties (yield and tensile strength) at different temperatures.

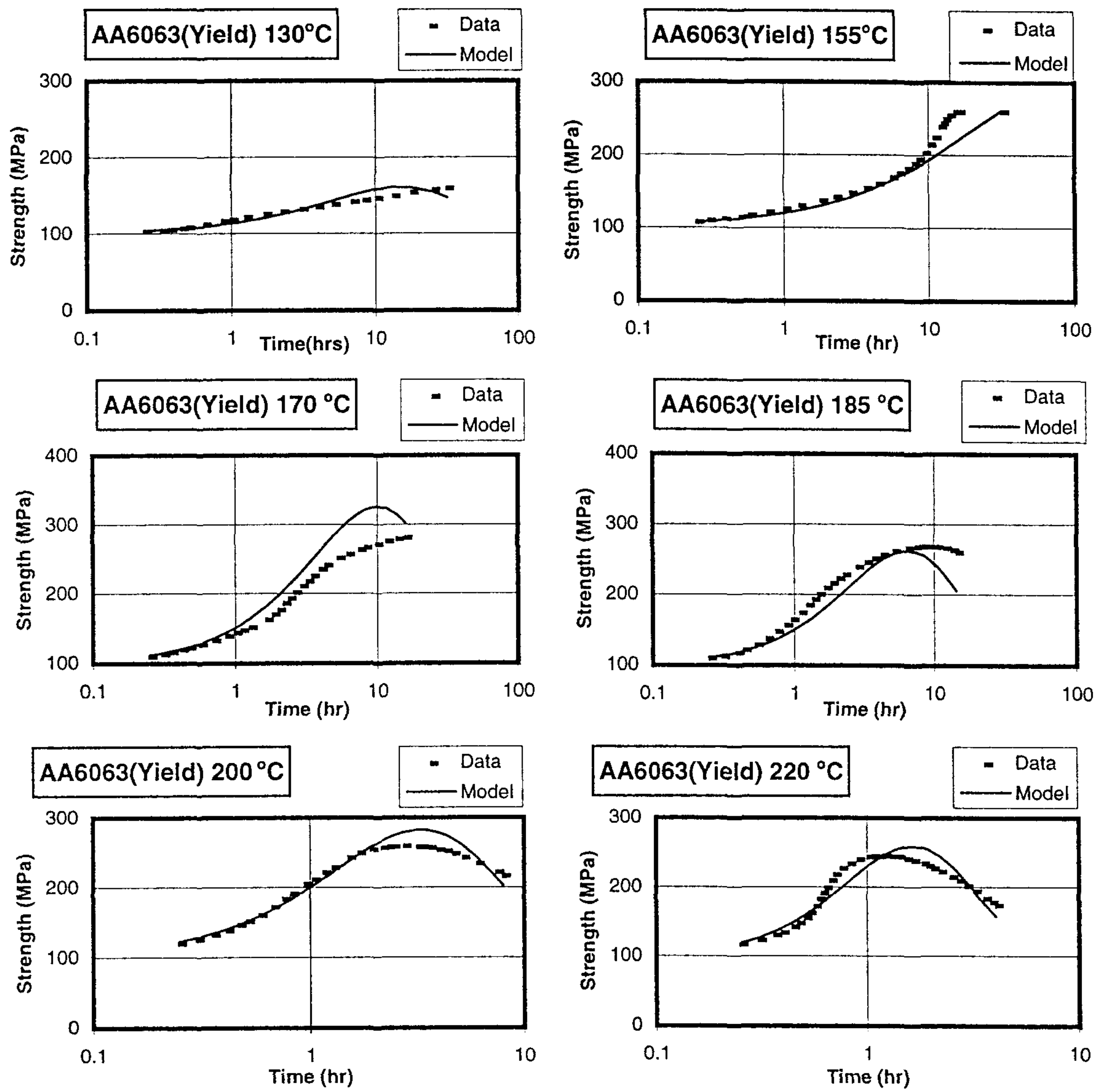

Fig. 3 Comparison of data and model after stage 2 regression (yield strength)

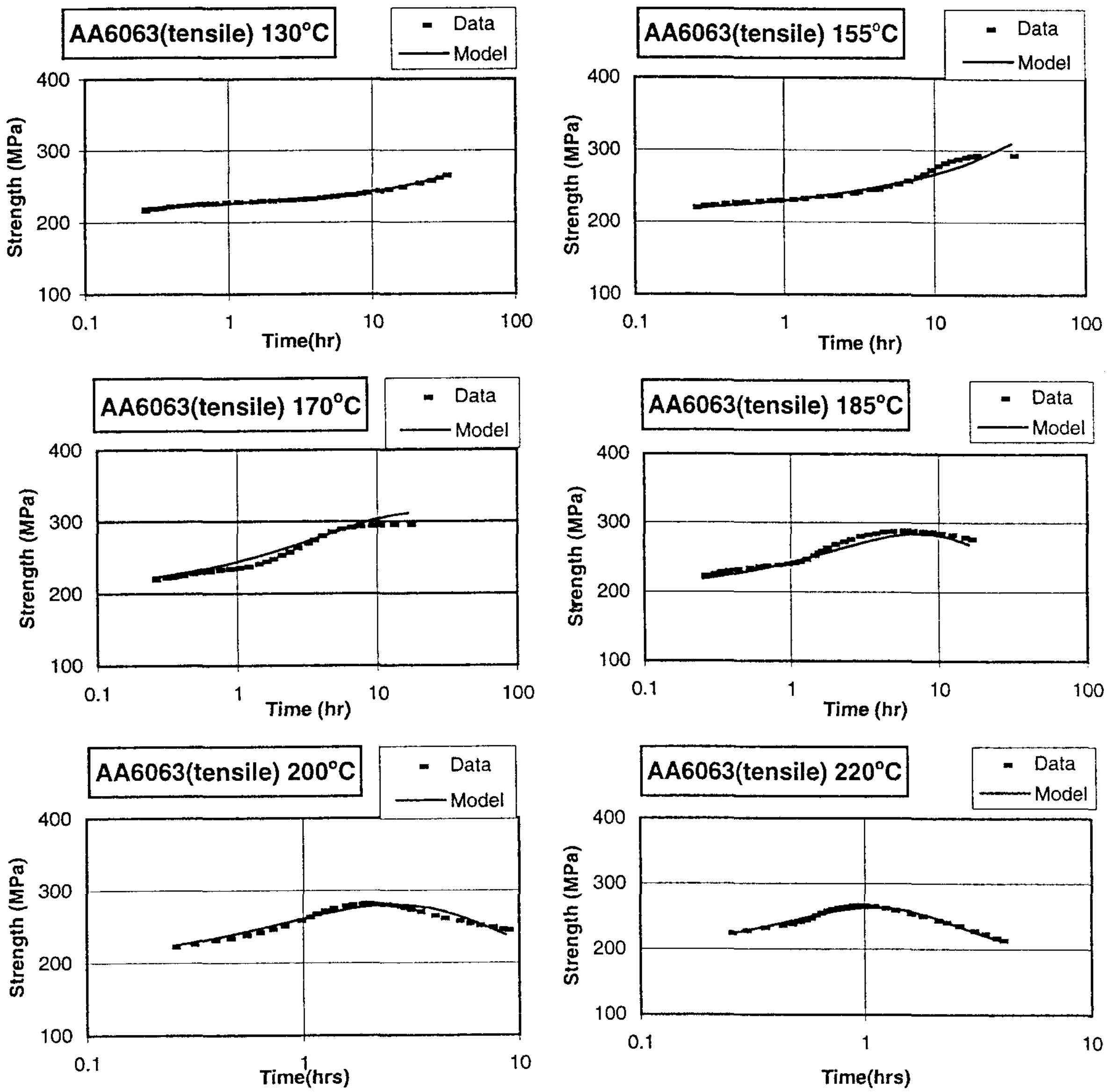

Fig. 4 Comparison of data and model after stage 2 regression (tensile strength)

Table 4 Values of subcoefficients for yield strength

Coefficient	AA6061 (σ_o = 150 MPa)	AA6063 (σ_o = 100 MPa)	AA6069 (σ_o = 300 MPa)
a_0	–21,542,185.77	–2,903,262.032	–494,812.113
a_1	–137,831.892	–17,884.17678	–1,360.0909
a_2	370,067,895	52,014,319.8	14,868,302.7
a_3	3,067,375.712	405,414.9008	48,977.1270
a_4	96.9930468	12.16976246	0
b_0	–2,315.9608	–9,264.405236	11,737.8485
b_1	–12,968,563	–54.25067475	32.836643
b_2	44,843.8445	174,065.2109	–347,968.76
b_3	309.026655	1,261.8533294	–1,171.3074
b_4	0.00791005	0.035008216	0
c_0	395.977417	5,434.95965	–1,064.2579
c_1	2.6364364	32.78772658	–2.9667816
c_2	–7,224.383587	–99,558.63189	31,683.32
c_3	–56.6284669	–750.8908579	106.005263
c_4	–0.00221539	–0.021904789	0

Table 5 Values of subcoefficients for tensile strength

Coefficient	AA6061 (σ_o = 250 MPa)	AA6063 (σ_o = 200 MPa)	AA6069 (σ_o = 400 MPa)
a_0	–15,291,826.54	–1,193,375.03	–900,900.108
a_1	–97,756.1	–7,361.34266	–2,517.05754
a_2	262,895,580	21,351,728.99	26,765,664.16
a_3	2,176,479	166,760.2804	89,839.6415
a_4	68.72475	5.016044415	0
b_0	28.7248	–2,325.7525	14,912.359
b_1	0.46855	–14.0865874	41.5343
b_2	347.667	42,380.39679	–444,470.310
b_3	–7.32422	321.995811	–1,484.66667
b_4	–0.00048	0.009459402	0
c_0	–566.792	3,235.215266	–763.36915
c_1	–2.20866	20.91346326	–2.09931
c_2	12,687.13	–58,541.7721	23,042.1238
c_3	65.95429	–468.973564	75.518866
c_4	0.000239	–0.01458596	0

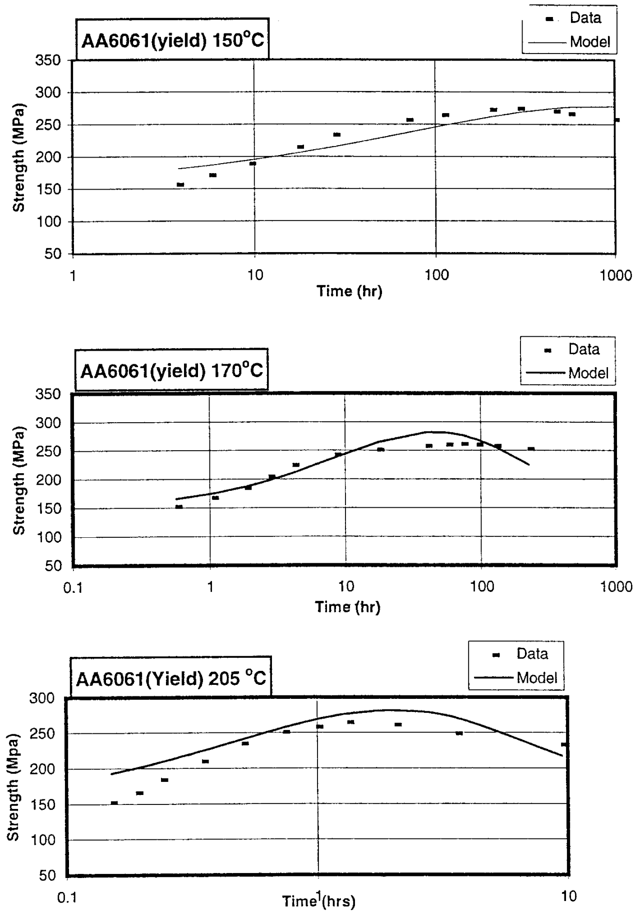

Fig. 5 Comparison of data and model after stage 2 regression (yield strength)

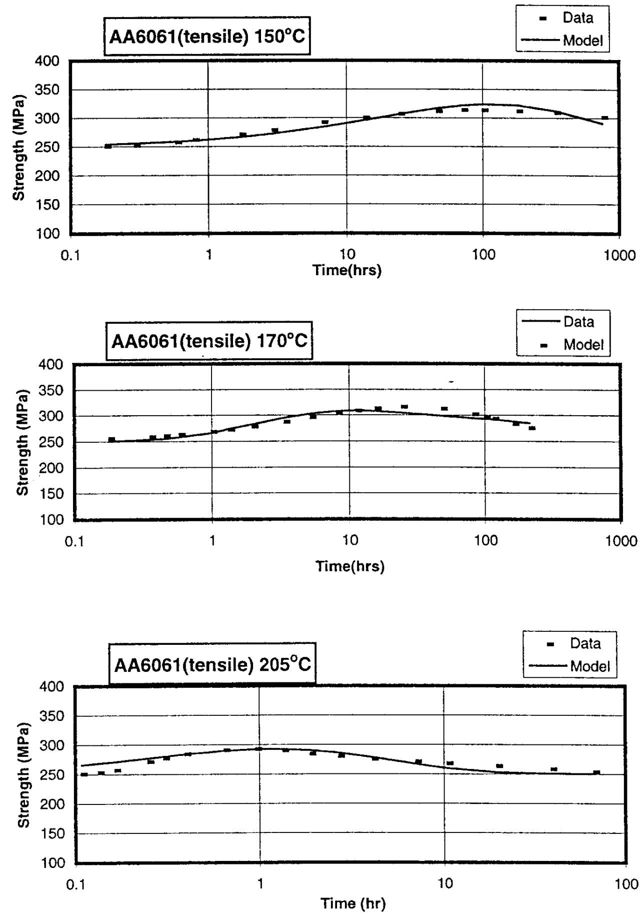

Fig. 6 Comparison of data and model after stage 2 regression (tensile strength)

5. Conclusions

A mathematical model based on experimental data has been developed to predict the mechanical-property (tensile strength and yield strength) response of a class of extrudable aluminum alloys as a result of age-hardening process. The aging temperature (T) and the residence time (t) at the aging temperature have been used as independent variables. It is shown that the model provides reasonably good prediction of mechanical-property dependence on independent variables (t and T) when compared to experimental data. A significant contribution is the use of the model to predict the mechanical properties at intermediate temperatures for which experimental data are not available. Additionally, the model can be successfully used in a computer-controlled aging environment. The model can also be used to provide a basis for writing computer codes for an aging process when a desired mechanical property is known and a best combination of independent variables (t and T) is sought.

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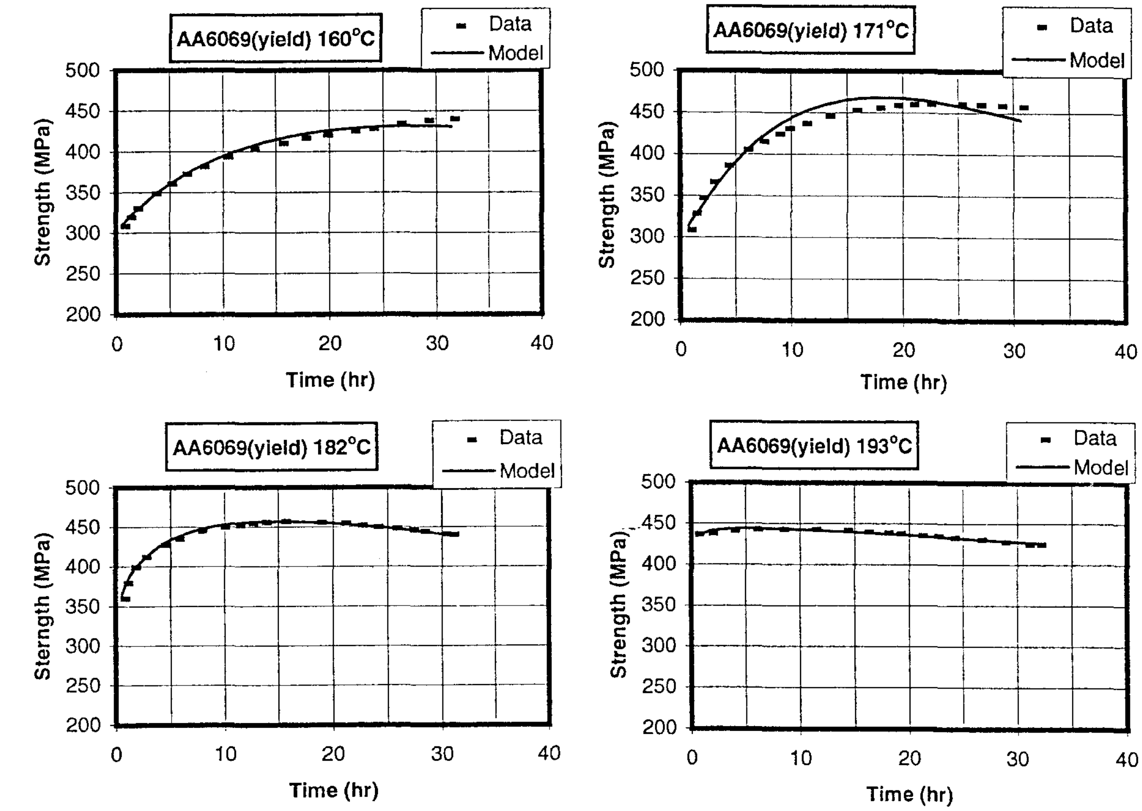

Fig. 7 Comparison of data and model after stage 2 regression (yield strength)

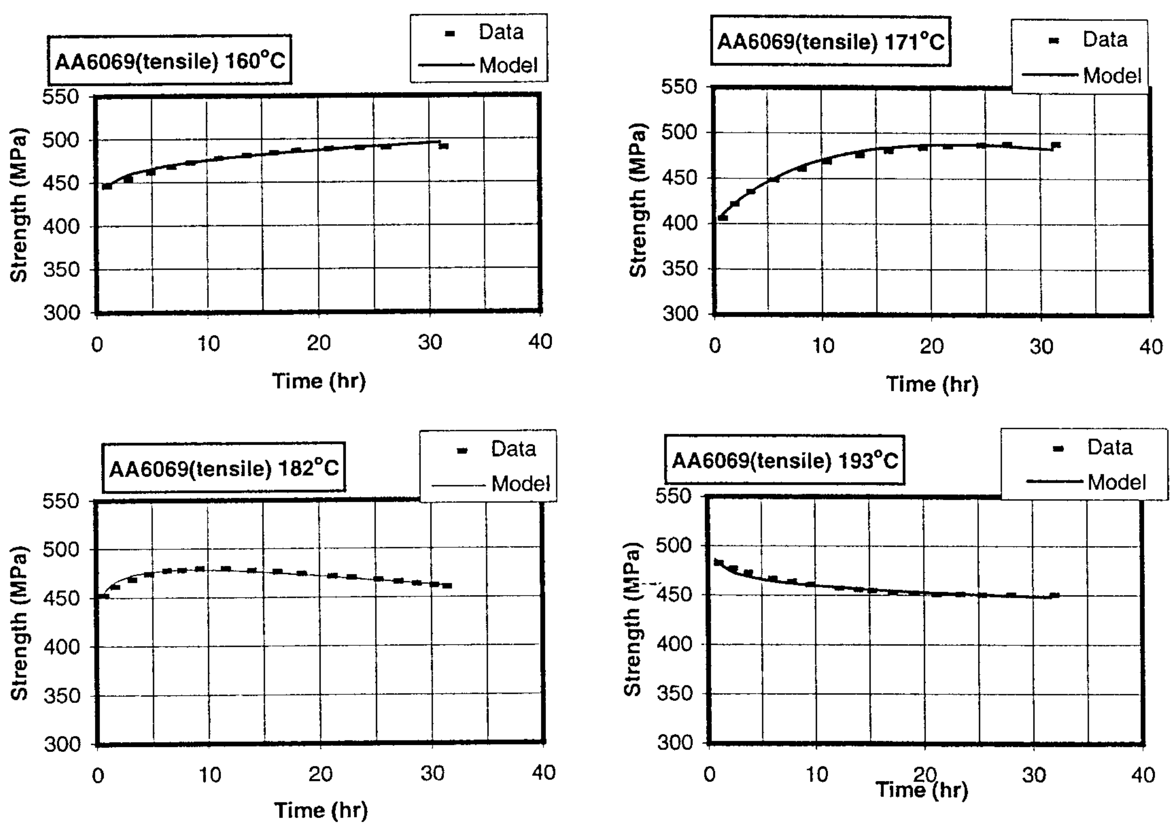

Fig. 8 Comparison of data and model after stage 2 regression (tensile strength)